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Full Length Research Paper

Assessing the Adequacy of Parameter Estimation Method of Gumbel Distribution for Modelling Annual 1-day Maximum Rainfall

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Estimation of design flood for a desired return period is of utmost importance for planning, design and management of hydraulic structures at the project site. This can be achieved by fitting of probability distribution to the recorded Annual 1-day Maximum Rainfall (AMR) data. This paper illustrates the use of Gumbel distribution for modelling AMR recorded at Fatehabad, Hansi, Hissar and Tohana rain-gauge stations. Parameter estimation methods such as method of moments, maximum likelihood method, Method of Least Squares (MLS), Mixed moments (MIX), order statistics approach, principle of maximum entropy and probability weighted moments are used for determination of distributional parameters. Kolmogorov-Smirnov (KS) test is used for checking the adequacy of fitting Gumbel distribution to the recorded data. Model Performance Indicators (MPIs) such as coefficient of determination and root mean square error are used for the selection of a suitable method for modelling AMR. The MPIs results show that the MLS is better suited for modelling AMR for Fatehabad, Hissar and Tohana, and MIX for Hansi; though KS test supports the use of all seven methods for determination of parameters of Gumbel for the stations under study.

Keywords: Gumbel Distribution, Kolmogorov-Smirnov Test, Method of Least Squares, Rainfall Analysis

INTRODUCTION

Estimation of design flood for a desired return period is of utmost importance for planning, design and operation of various hydraulic structures such as dams, bridges, barrages and storm water drainage systems, etc. These include different types of flood such as standard project flood, probable maximum flood and design basis flood (IAEA, 1981). In case of large river basins, the hydrological and streamflow series of a significant duration are generally available. However, for ungauged basins, enough data is not available other than rainfall of shorter duration pertaining to a neighbouring basin (WMO, 1994). Rainfall depth thus becomes an important input in derivation of flood discharge. Depending on the size and the proposed life of the structure, the estimated rainfall corresponding to a desired return period is used. In this context, Atomic Energy Regulatory Board (AERB, 2008) described that the 1000-year (yr) return period Mean+SE (where Mean denotes the estimated rainfall and SE the Standard Error) is generally considered to arrive at a design parameter that a structure must withstand during its lifetime For arriving at such design values, frequency analysis is identified as an effective and expedient tool for modelling AMR data (Casas et al., 2011). The procedure enables estimation of the probability of occurrence of a certain hydrological event of practical importance by fitting a probability distribution to the recorded rainfall data.

Research studies described that the Gumbel distribution is generally adopted for modelling rainfall data to arrive at a design parameter for a station or region though number of distributions such as Gamma, Normal, Lognormal, Pearson Type-III, Log Pearson Type-III and Weibull are available for rainfall frequency analysis (IAEA, 2003); and therefore Gumbel distribution is considered in the present study. Mujere (2011) applied Gumbel distribution for modelling of flood data for Nyanyadzi River, Zimbabwe. Baratti et al. (2012) carried out flood frequency analysis at seasonal and annual time scales for Blue Nile River adopting Gumbel distribution. Esteves (2013) applied extreme value distribution to estimate the extreme precipitation depths at different rain-gauge stations in southeast United Kingdom. Standard analytical procedures like Method of Moments (MOM), Maximum Likelihood Method (MLM), Method of Least Squares (MLS), Mixed Moments (MIX), Order Statistics Approach (OSA), Principle of Maximum Entropy (PME) and Probability Weighted Moments (PWM) are applied for determination of parameters of Gumbel distribution (Arora and Singh, 1987). Number of studies has been carried out by different researchers on analyzing the characteristics of the parameter estimation methods of Gumbel distribution (Landwehr et al., 1979; Phien, 1987; Ranyal and Salas, 1986; Swami et al., 1986; and Rasmussen and Gautam, 2003). But there is no general agreement in applying particular method for determination of parameters of Gumbel distribution for estimation of rainfall for a station or region. In this paper, an attempt has been made to compare the seven parameter estimation methods of Gumbel distribution for modelling AMR recorded at Fatehabad, Hansi, Hissar and Tohana rain-gauge stations. Kolmogorov-Smirnov (KS) test is employed for checking the adequacy of fitting of the method to the recorded AMR data. Model Performance Indicators (MPIs) such as coefficient of determination (R^2) and root mean square error (RMSE) are used for selection of a suitable method of Gumbel distribution for modelling AMR data for the stations under study. The procedures adopted in modelling AMR, computation of KS statistic value and MPIs by different methods of Gumbel distribution are described in the ensuing sections.

METHODOLOGY

The Probability Density Function [PDF; f(R)] and Cumulative Distribution Function [CDF; F(R)] of Gumbel distribution is given by:

$$f(R) = \frac{e^{-(R_i-\alpha)/\beta} e^{-e^{-(R_i-\alpha)/\beta}}}{\beta}, \ \beta>0 \quad (1)$$

$$F(R) = e^{-e^{-(R_i-\alpha)/\beta}}, \ \beta>0 \quad (2)$$

where α and β are the location and scale parameters of the distribution (Gumbel, 1960). The parameters are computed by seven methods; and used to estimate the rainfall (R_T) for different return periods from $R_T = \alpha + Y_T\beta$, where $Y_T = -\mathrm{Ln}(-\mathrm{Ln}(1-(1/T)))$. The procedures involved in determining the parameters of Gumbel by different methods are as follows:

Parameter Estimation Methods

Method of Moments

$$\alpha = \overline{R} - 0.5772157\beta \text{ and } \beta = (\sqrt{6}/\pi)S_R \quad (3)$$

where and S_R are the average value and standard deviation of the recorded AMRs (Gumbel, 1960).

Maximum Likelihood Method

$$\beta = \overline{R} - \left[\sum_{i=1}^{N} R_i \exp(-R_i/\beta) \middle/ \sum_{i=1}^{N} \exp(-R_i/\beta)\right] \text{ and}$$

$$\alpha = -\beta \mathrm{Ln}\left[\sum_{i=1}^{N} \exp(-R_i/\beta) \middle/ N\right] \quad (4)$$

where R_i is the recorded AMR of i^{th} sample and N is the sample size (Lee and Heo, 2011).

Method of Least Squares

$$\beta = \left(\left(\sum_{i=1}^{N} R_i\right)^2 - \left(N\sum_{i=1}^{N} R_i^2\right)\right) \middle/ \left(\left(N\sum_{i=1}^{N} R_i(\mathrm{Ln}(-\mathrm{Ln}(P_i)))\right) - \left(\sum_{i=1}^{N} R_i\right)\left(\sum_{i=1}^{N} \mathrm{Ln}(-\mathrm{Ln}(P_i))\right)\right)$$

$$\alpha = \overline{R} + \left(\left(\sum_{i=1}^{N} \mathrm{Ln}(-\mathrm{Ln}(P_i))\right)\beta \middle/ N\right) \quad (5)$$

where P_i=(m-0.44)/(N+0.12) and Ln(-Ln(P_i)) defines the cumulative probability of non-exceedance for each R_i (Manik and Datta, 1998). Here 'm' is the rank assigned to each sample arranged in ascending order (i.e., rank '1' is assigned to the smallest value of the sample and 'N' to the largest value)

Mixed Moments

$$\beta = S_R/1.28255 \text{ and}$$

$$\alpha = \beta \mathrm{Ln}\left(1 + (1/\beta)\overline{R} + (1/2\beta^2)\sum_{i=1}^{N}\left(R_i^2 \middle/ N\right)\right) \quad (6)$$

Other Statistics Approach

The parameters of Gumbel distribution are given by:

$$\alpha = r^*\alpha_M^* + r'\alpha_M' \text{ and } \beta = r^*\beta_M^* + r'\beta_M' \quad (7)$$

where r^* and r' are proportionality factors, which can be obtained from the selected values of k, n and n′ using the relations as follows:

$$r^* = kn/N \text{ and } r' = n'/N$$

where N is the sample size containing the basic data, which can be written as N=kn+n′ (Here, n is the number of elements in each sub group (k) and n′ is the remainders). α_M^* and β_M^* are the distribution parameters of the groups, and α_M' and β_M' are the parameters of the

Table 1: Weights α_{ni} and β_{ni} for determination of parameters (using OSA)

α_{ni} (or) β_{ni}	i					
	1	2	3	4	5	6
α_{2i}	0.91637	0.08363				
α_{3i}	0.65632	0.25571	0.08797			
α_{4i}	0.51099	0.26394	0.15368	0.07138		
α_{5i}	0.41893	0.24628	0.16761	0.10882	0.05835	
α_{6i}	0.35545	0.22549	0.16562	0.12105	0.08352	0.04887
β_{2i}	-0.72135	0.72135				
β_{3i}	-0.63054	0.25582	0.37473			
β_{4i}	-0.55862	0.08590	0.22392	0.24879		
β_{5i}	-0.50313	0.00653	0.13046	0.18166	0.18448	
β_{6i}	-0.45927	-0.03599	0.07319	0.12672	0.14953	0.14581

remainders (Lieblein, 1974). These can be computed from the following equations:

$$\alpha_M^* = (1/k)\sum_{i=1}^{n}\alpha_{ni}S_i;\ \alpha_M^{'} = \sum_{i=1}^{n'}\alpha_{n'i}R_i;$$

$$\beta_M^* = (1/k)\sum_{i=1}^{n}\beta_{ni}S_i \text{ and } \beta_M^{'} = \sum_{i=1}^{n'}\beta_{n'i}R_i \quad (8)$$

where $S_i = \sum_{i=1}^{n} R_{ij}$, j=1,2,3,..,k. Here, R_i is the i^{th} sample in the remainder group having n' elements, R_{ij} is the i^{th} sample in the j^{th} group having n elements. Table 1 gives the weights of α_{ni} and β_{ni} used in determination of parameters of Gumbel distribution.

Principle of Maximum Entropy

$$\alpha = -\beta \mathrm{Ln}\left[\sum_{i=1}^{N}\exp(-R_i/\beta)\Big/N\right] \text{ and}$$

$$\beta = \overline{R}\Big/\left(0.5772157 + \mathrm{Ln}\left(N\Big/\sum_{i=1}^{N}\exp(-R_i/\beta)\right)\right) \quad (9)$$

Probability Weighted Moments

$$\alpha = M_{100} - (0.5772157)\beta \text{ and } \beta = (M_{100} - 2M_{101})/\ln 2 \quad (10)$$

where $M_{100} = \overline{R}$ and $M_{101} = \sum_{i=1}^{N} R_i(N-i)/(N(N-1))$ (Arora and Singh, 1987).

Computation of Standard Error

Standard Error (SE) on the estimated rainfall adopting Gumbel distribution (using seven methods) may be computed from the following equation:

$$SE = \frac{\beta}{\sqrt{N}}\left(A + BY_T + CY_T^2\right)^{0.5} \quad (11)$$

The coefficients used in computation of SE by different methods are given in Table 2.

Table 2: Coefficients used in estimation of SE by different methods

Parameter estimation method	Coefficients used in computation of SE		
	A	B	C
MOM, PME and MLS	1.1589	0.1919	1.1000
PWM	1.1128	0.4574	0.8046
MLM	1.1087	0.5140	0.6079
MLX	1.1700	0.1960	1.0990
MOM: Method of Moments; MLM: Maximum Likelihood Method; MLS: Method of Least Squares; MIX: Mixed Moments; PME: Principle of Maximum Entropy; PWM: Probability Weighted Moments			

The values of SE on the estimated rainfall by OSA can be obtained from

$$SE = \left(r^* R_n + r' R_{n'}\right)^{1/2} \quad (12)$$

where $r^* = (1/k)(kn/N)^2$ and $r' = \left(n'/N\right)^2$. R_n and $R_{n'}$ are defined by the general form as

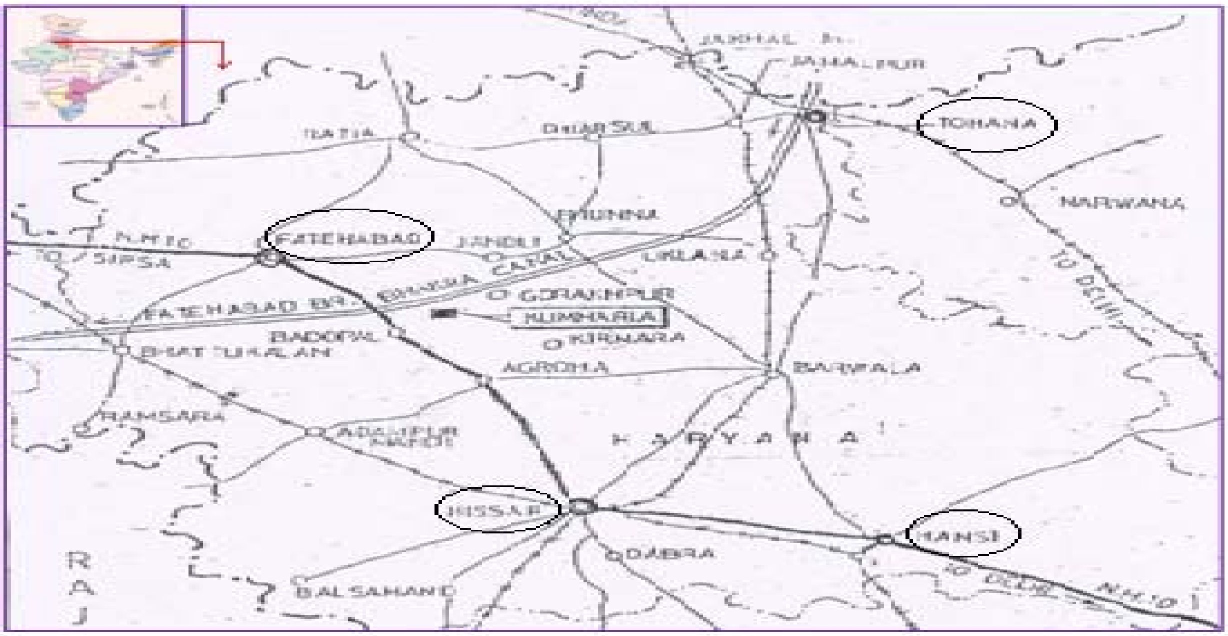

Figure 1: Location map of the rain-gauge stations
(Inside figure shows the location of Haryana)

$R_n = \left(A_n Y_T^2 + B_n Y_T + C_n\right)\beta^2$. The values of A_n, B_n, and C_n used in computing the SE for OSA, are given in Table 3 (AERB, 2008).

Table 3: Variance determinators for R_n

n	A_n	B_n	C_n
2	0.71186	-0.12864	0.65955
3	0.34472	0.04954	0.40286
4	0.22528	0.06938	0.29346
5	0.16665	0.06798	0.23140
6	0.13196	0.06275	0.19117

Kolmogorov-Smirnov Test

The KS statistic is defined by

$$KS = \underset{i=1}{\overset{N}{Max}}\left(F_e\left(R_i\right) - F_D\left(R_i\right)\right) \tag{13}$$

Here $F_e(R_i)$=(i-0.44)/(N+0.12) is the empirical CDF of R_i and $F_D\left(R_i\right)$ is the computed CDF of R_i. If the computed value of KS statistic is less than that of theoretical value at the desired significance level, then the method is considered to be acceptable for determination of parameters of Gumbel distribution for modelling AMR data (Zhang, 2002).

Model Performance Indicators

Theoretical descriptions of R^2 and RMSE are given by

$$R^2 = \left(\frac{\sum_{i=1}^{N}\left(R_i - \overline{R}\right)\left(R_i^* - \overline{R^*}\right)}{\sqrt{\left(\sum_{i=1}^{N}\left(R_i - \overline{R}\right)^2\right)\left(\sum_{i=1}^{N}\left(R_i^* - \overline{R^*}\right)^2\right)}}\right)^2 \tag{14}$$

$$RMSE = \left(\left(1/N\right)\left(\sum_{i=1}^{N}\left(R_i - R_i^*\right)^2\right)\right)^{0.5} \tag{15}$$

where R_i and R_i^* are the recorded and estimated AMRs of i[th] sample, $\overline{R^*}$ is the average value of the estimated AMRs. The method having the better R^2 and minimum RMSE is considered as the most suitable method for modelling AMR data (Vijayagopal et al., 2013).

APPLICATION

An attempt has been made to fit the AMR data recorded at Fatehabad, Hani, Hissar and Tohana rain-gauge stations using seven methods of Gumbel distribution. The rain-gauge stations are located at a distance of about 17 km, 34 km, 43 km, and 50 km from Gorakhpur, Haryana. Figure 1 shows the location map of the rain-gauge stations considered in the study.

The series of AMR was retrieved from daily rainfall data and used for Extreme Value Analysis (EVA) of rainfall. Daily rainfall data recorded at Fatehabad and Hansi for the period 1954-2005, Hissar for the period 1969-2007 and Tohana for the period 1951-2005 are

Table 4: Statistical parameters of AMRs

Rain-gauge Station	Statistical parameters of AMR				
	$\overline{R}$ (mm)	S_R (mm)	CV	Skewness	Kurtosis
Fatehabad	64.7	31.6	0.489	0.286	0.722
Hansi	356.2	163.2	0.458	-0.179	0.288
Hissar	94.8	59.2	0.624	1.779	1.520
Tohana	72.8	39.4	0.542	0.120	0.960

$\overline{R}$: Average value of recorded AMRs; S_R: Standard Deviation; CV: Coefficient of Variation

used. Table 4 gives the statistical parameters of AMRs recorded at the stations under study.

RESULTS AND DISCUSSION

By applying the procedures described above, a computer program was developed and used to fit the AMR data recorded at Fatehabad, Hansi, Hissar and Tohana stations. The program computes the distributional parameters, rainfall estimates for different return periods, KS statistic values and MPIs. Figures 2-5 show the plots of recorded and estimated AMRs for Fatehabad, Hansi, Hissar and Tohana stations respectively.

From Figure 2, it can be seen that there is no significant difference between the estimated rainfalls adopting Gumbel distribution (using seven methods) for Fatehabad. Also, from Figures 3-5, it can be seen that there is some difference between estimated rainfalls by seven methods for Hansi, Hissar and Tohana. Therefore, KS test and MPIs are used to identify the most suitable method for modelling AMR data for the stations under study.

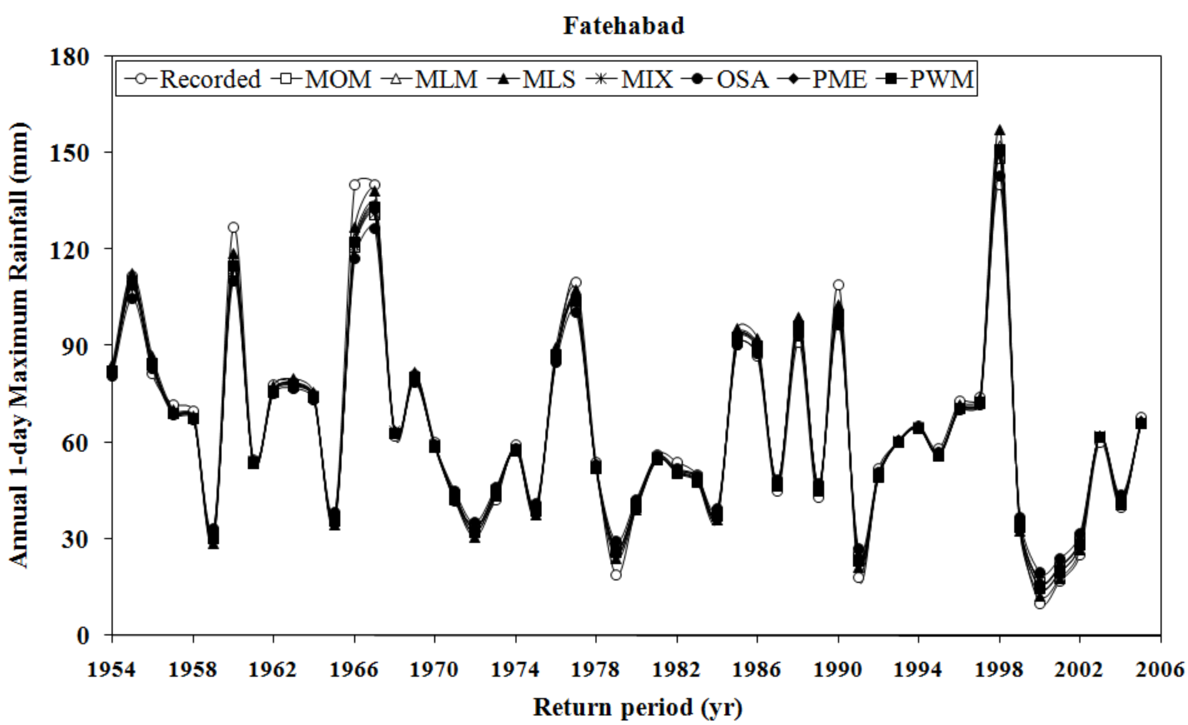

Figure 2: Plot of recorded and estimated AMRs for the period 1954-2005 for Fatehabad

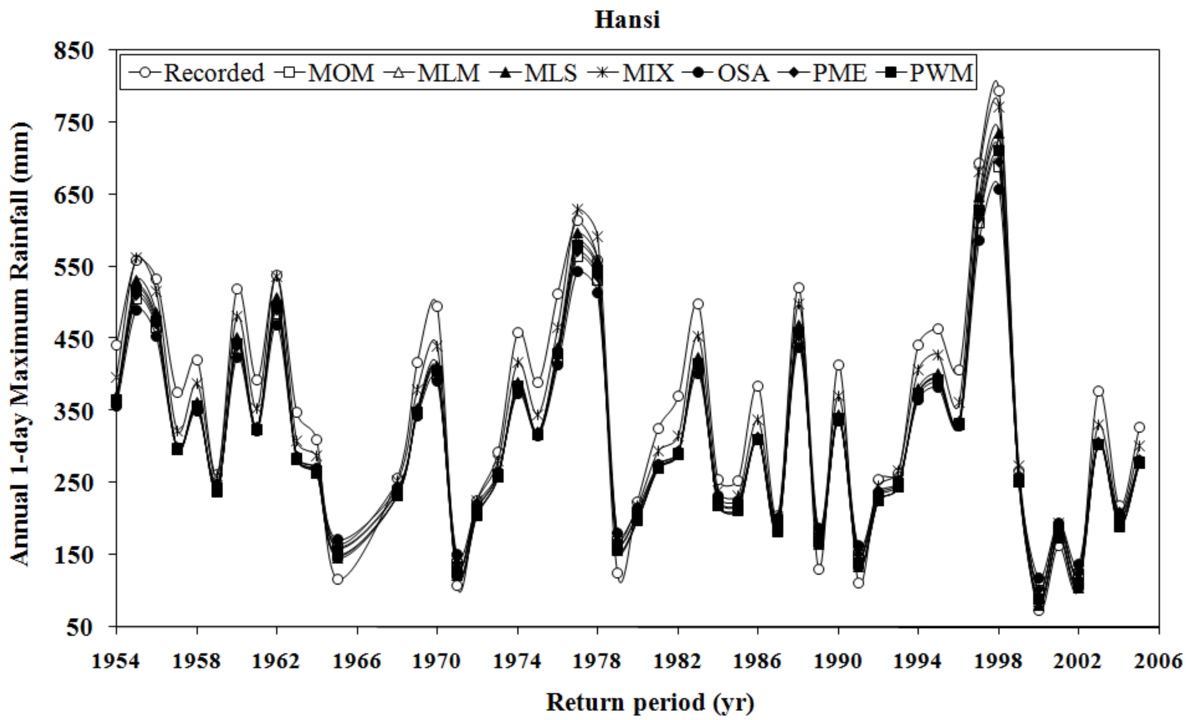

Figure 3: Plot of recorded and estimated AMRs for the period 1954-2005 for Hansi

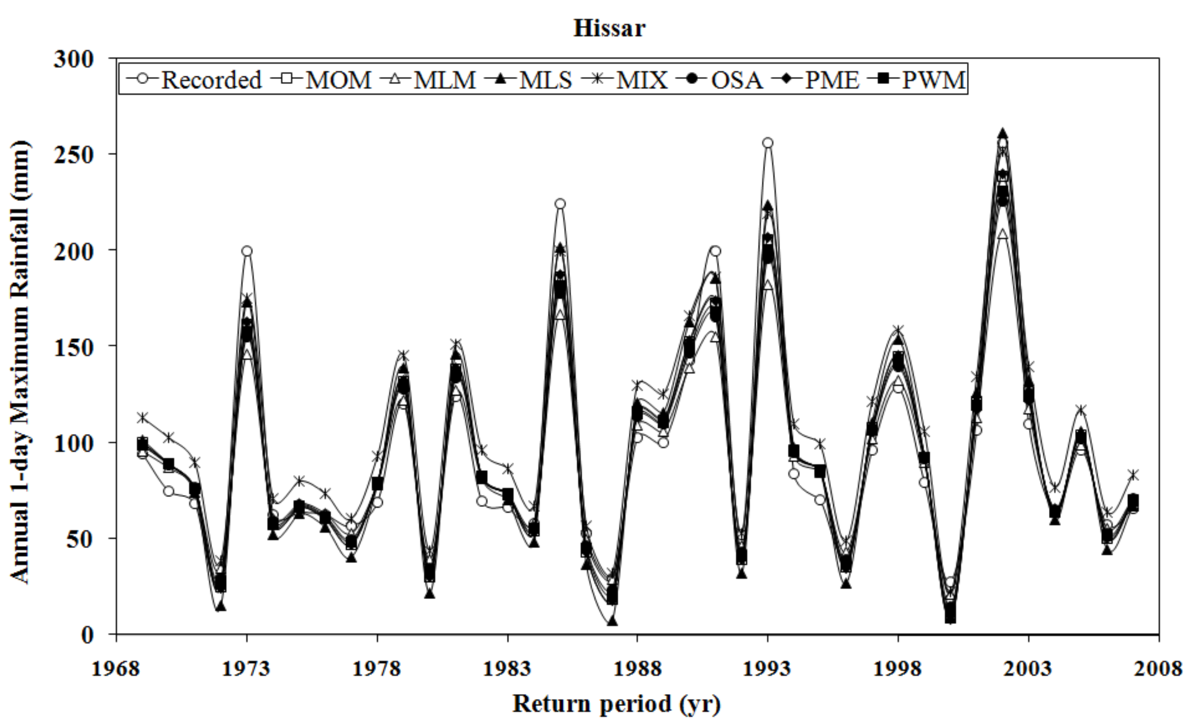

Figure 4: Plot of recorded and estimated AMRs for the period 1969-2007 for Hissar

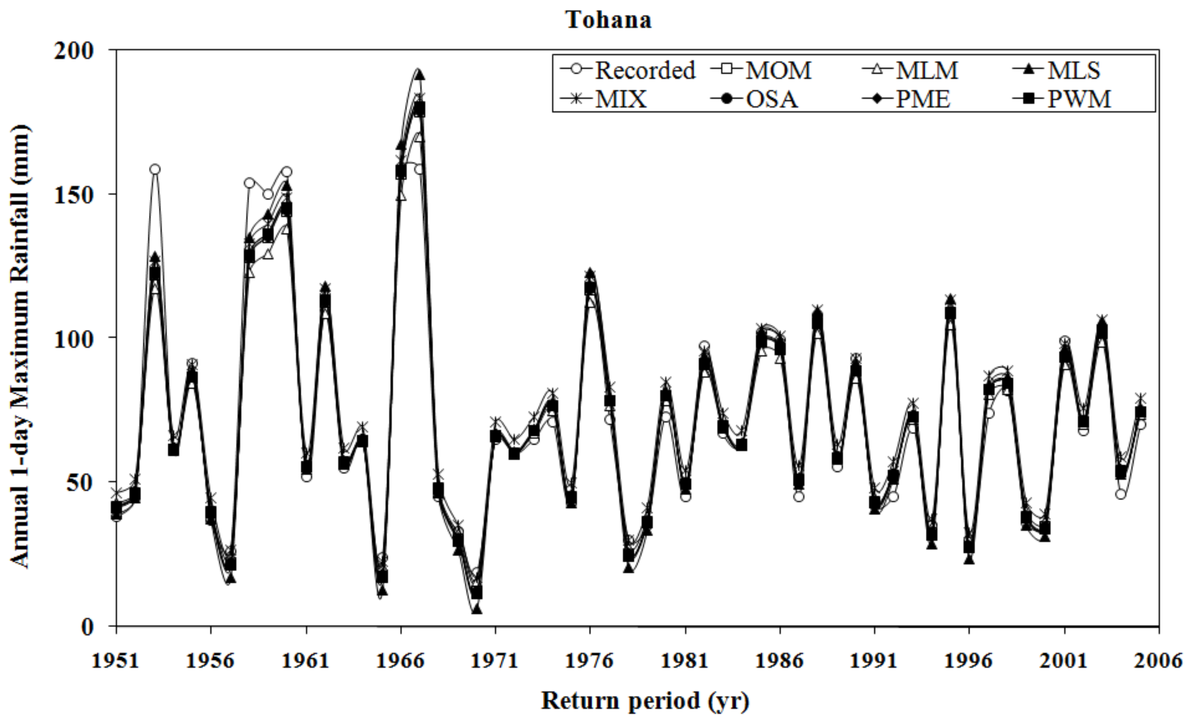

Figure 5: Plot of recorded and estimated AMRs for the period 1951-2005 for Tohana

Table 5: Computed values of KS statistic by seven methods of Gumbel distribution

Rain-gauge Station	Computed values of KS statistic using						
	MOM	MLM	MLS	MIX	OSA	PME	PWM
Fatehabad	0.064	0.067	0.069	0.058	0.066	0.056	0.069
Hansi	0.126	0.112	0.104	0.103	0.142	0.119	0.116
Hissar	0.125	0.136	0.157	0.215	0.132	0.123	0.128
Tohana	0.094	0.105	0.091	0.146	0.095	0.099	0.089

Analysis Based on KS Test

For the assessment on fitting of Gumbel distribution (using seven methods) to the recorded AMR data series, KS statistic values are computed from Eq. (13), and given in Table 5.

From Table 5, it may be noted that the computed values of KS statistic from Gumbel (using seven parameter estimation methods) distribution are less than the theoretical values (0.189 for Fatehabad and Hansi, 0.218 for Hissar and 0.183 for Tohana) at 5% significance level, and hence at this level, all seven methods of Gumbel are considered to be acceptable to fit the AMR data recorded at the stations under study.

Analysis Based on MPIs

The performance of the methods of Gumbel distribution used for modelling AMR was adjudged by R^2 and RMSE. Table 6 gives the values of R^2 and RMSE computed by seven methods through Eqs. (14) and (15) for the stations under study.

Based on R^2 values, it may be noted that there is good agreement between the methods used for modelling AMR data for the stations under study; and there is no difference between the R^2 values. The R^2 values given by seven methods of Gumbel distribution are noted to be 0.982 for Fatehabad, 0.970 for Hansi, 0.926 for Hissar and 0.957 for Tohana. From Table 6, it may be noted that the RMSE values given by MLS are relatively minimum when compared to the corresponding values of other six methods of Gumbel distribution for Fatehabad, Hissar and Tohana. Also, from Table 6, it may be noted that the MIX provides minimum RMSE while modelling AMR data for Hansi. Based on RMSE values, MLS is identified as better suited for determination of distributional parameters for modelling AMR data recorded at Fatehabad, Hissar and Tohana; and MIX for Hansi. Table 7 gives the rainfall

Table 6: R^2 and RMSE values computed by seven methods of Gumbel distribution

Rain-gauge Station	Coefficient of Determination (R^2)							RMSE (mm)						
	MOM	MLM	MLS	MIX	OSA	PME	PWM	MOM	MLM	MLS	MIX	OSA	PME	PWM
Fatehabad	0.982	0.982	0.982	0.982	0.982	0.982	0.982	4.8	4.3	4.2	4.8	5.9	4.6	4.5
Hansi	0.970	0.970	0.970	0.970	0.970	0.970	0.970	57.5	52.4	49.2	32.6	64.0	54.7	54.1
Hissar	0.926	0.926	0.926	0.926	0.926	0.926	0.926	16.4	21.0	16.1	20.6	17.9	16.3	17.1
Tohana	0.957	0.957	0.957	0.957	0.957	0.957	0.957	8.2	9.2	8.1	9.1	8.3	8.2	8.3

Table 7: Rainfall estimates together with SEs for different return periods for Fatehabad, Hansi, Hissar, and Tohana stations.

Return period (yr)	Estimated rainfall (mm) together with SE (mm) for							
	Fatehabad (using MLS)		Hansi (using MIX)		Hissar (using MLS)		Tohana (using MLS)	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE
2	59.8	4.4	319.2	29.4	85.4	10.0	66.5	5.4
5	90.6	7.5	463.4	49.5	145.8	16.9	105.4	9.2
10	110.9	10.1	558.9	66.8	185.8	22.8	131.1	12.4
20	130.5	12.7	650.5	84.4	224.1	28.9	155.8	15.6
50	155.8	16.3	769.0	107.8	273.8	36.9	187.7	30.0
100	174.8	19.0	857.8	125.6	311.0	42.9	211.6	23.3
200	193.7	21.7	946.4	143.4	348.0	49.0	235.5	26.6
500	218.6	25.2	1063.1	167.0	396.9	57.1	267.0	30.9
1000	237.5	28.0	1151.4	185.0	434.0	63.3	290.7	34.1

estimates together with standard errors for different return periods vary from 2-yr to 1000-yr for the stations under study.

From the results of EVA of rainfall, it is suggested that the 1000-yr return period Mean+SE (where Mean denotes the estimated rainfall (R_{1000}) and SE the Standard Error) values of about 266 mm, 1336 mm, 497 mm and 325 mm may be adopted for designing of hydraulic structures in Fatehabad, Hansi, Hissar and Tohana stations respectively.

CONCLUSIONS

The paper described a computer aided procedures for modelling AMR data recorded at Fatehabad, Hansi, Hissar and Tohana rain-gauge stations adopting Gumbel distribution (using MOM, MLM, MLS, MIX, OSA, PME and PWM). From the results of the data analysis, the following conclusions are drawn from the study.

i) KS test results confirm the use of all seven methods of Gumbel for modelling AMR data for the stations under study.

ii) R^2 results indicated that there is a good agreement between the methods used for modelling AMR data recorded at all four stations. The R^2 values are noted to be 0.982 for Fatehabad, 0.970 for Hansi, 0.926 for Hissar and 0.957 for Tohana.

iii) The RMSE values showed that the MLS is better suited amongst seven methods for determination of parameters of Gumbel distribution for Fatehabad, Hissar and Tohana; and MIX for Hansi.

iv) RMSE on the estimated rainfalls using MLS, with reference to the recorded rainfalls, is about 4 mm for Fatehabad, 16 mm for Hissar and 8 mm for Tohana. For Hansi, the RMSE is computed as about 33 mm while MIX is considered for parameter estimation for modelling AMR.

v) The 1000-yr return period Mean+SE (where Mean denotes the estimated rainfall and SE the Standard Error) values of about 266 mm, 497 mm, 325 mm given by Gumbel distribution (using MLS) may be considered for designing of hydraulic structures in Fatehabad, Hissar and Tohana stations respectively.

vi) For Hansi, the 1000-yr return period Mean+SE value of 1336 mm may be considered for design purposes in Hansi while MIX is used for modelling AMR data.

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